Introduction to mathematics

Part I: Presentation, outline and motivations

Contents

Introduction

The documents from this series are meant as an introduction to the essence of mathematics for people without much prior knowledge in math: no assumptions on what is already known to the reader are made. However, understanding the concepts and motivations exposed in those documents is significantly easier for the reader that is already familiarised with some basic results in math or, more generally, that has studied in a scientific field requiring the use of mathematics. Nonetheless, we will put all our efforts in making this content as accessible as possible.

This is a rather informal and intuition-based introduction to mathematics whose goal is to build some elementary knowledge of the inner workings of math as to be able to learn more advanced math subjects afterwards. The main objective is to familiarise the reader with basic mathematical vocabulary, notations and methods. Traditionally the content exposed in those documents would be taught to first-year undergrad students.

Outline of the series

In part II, we will start by introducing the basic elements of **mathematical logic** such as propositions, logical operators and quantifiers. Then we will discuss the main **methods of proof** used in math.

In part III, we will concern ourselves with elementary notions in **set theory** and talk about the well-known **sets of numbers**.

In part IV, we will examine the concepts of relations, of **functions** (or maps) between sets, of **sequences** and the corresponding vocabularies.

In part V, we will present the major **fields of mathematics** in order to give directions on what to learn next.

What is and why is math?

To answer these questions we could take an historical approach, examine if and how developments of mathematics occurred in the past in different civilisations and draw conclusions as to why are mathematics a useful tool. However, even though this approach is well-suited to give motivations behind concepts and results in math, it reduces mathematics only to its applications and dismisses what we consider the real strength of math: **abstractions**.

In math, we do not concern ourselves with physical/material objects but rather abstractions, that is, pure mental representations of objects that do not require for our senses to detect them but rather our mind to understand. For instance, when we study lines in geometry we do not consider the line segment we draw with a pencil on a sheet of paper to represent the 'line' as the real object of our study but rather a mere representation, with its own limitations, of what we are actually manipulating, that is a infinite set of points with no width.

These abstractions, that we will from now on also call mathematical objects, are described by explicit unambiguous properties stating what is the essence of the object and how it behaves with other mathematical objects. We call those properties the **axioms** of the mathematical object. These are the 'atoms' of our mathematical reasoning: they are the assertions that we take for granted and that will act as the foundations for the development of our mathematical theories.

But how do we develop such theories? This is where the concept of **proof** enters the picture. If we would have to define what are mathematics, we would say that it is the science of deductive proof. A deductive proof is the process of applying deductive rules on propositions. We will take a closer look at what we mean by propositions in part II. Here, what we mean by deductive rules, in an informal manner, is the possible ways we can *infer* a proposition from another, that is, if the former is true then the latter also holds true.

All in all, a theory in math is like a game for which the rules are the deductive rules we use in our proofs and the axioms of our mathematical objects we study. It is like a jigsaw puzzle where the pieces are the mathematical objects and the shapes of the pieces are defined by the axioms and deductive rules. And math is all about the study of those games/puzzles: determine what holds true or false in those models, that is, in our analogy, which piece fit or not with whichever other piece.